We have also the following relation — more curious than useful :—

$$V_{x+n|m-n}=1-\frac{1-V_{x|m}}{1-V_{x|n}}.$$

Again,

$$1-V_{x|n}=\frac{1+a_{x+n}}{1+a_x}=\frac{\pi_x+d}{\pi_{x+n}+d}=\frac{1-A_{x+n}}{1-A_x},$$

formulæ which are very useful in many actuarial investigations.

Determination and Distribution of Profit. *By* JAMES MEIKLE, *Actuary of the Scottish Provident Institution.*

[Read before the Institute, 29th February, 1864, and printed in abstract by order of the Council.]

IN the paper which I had recently the pleasure of submitting to the Institute, I gave an explanation of what was meant by the phrase "the value of a policy." It was therein stated that, *in one view,* the value of a policy was the difference between the present value of the sum assured payable by the Office, and the present value of the premiums payable by the person assured; and *in another view,* that the value of a policy was the difference between the premiums received and the claims paid, accumulated at the rates of mortality and interest inherent in the calculation of the premiums. The *first* of these being a view of the value in respect of the *future* years; and the *second,* in respect of the *past* years. It was further stated, that so long as the experienced rates of mortality and interest coincided with the expected rates, the results brought out by these two methods of calculation would be the same. I shall now consider values of policies when the rates of mortality and interest which have been experienced are *different* from those inherent in the original calculations—that is to say, when different rates have been experienced in the past years from the rates which it was expected would have been experienced, or which it is expected will be experienced in the future years. I propose, in short, to consider the subject of profit; to trace its sources, and to state the modes in which it should be determined and distributed among the policy-holders.

The author remarks, that "few actuaries have written upon profit, the sources of its creation, and the modes of its distribution," and that those who have done so have not been sufficiently elemen-

tary in their investigations. He therefore wishes to supply this deficiency. He will show wherein his processes of investigating profit, and the results thereto deduced, differ from those of former writers; and he trusts that others who may dissent from his views will have no scruples in pointing out wherein they differ from him. He then proceeds :—

Disregarding the primary importance of ascertaining whence profit has been derived, Life Assurance Offices generally have first determined the manner and the nature of its distribution. * * * I propose to consider the nature of the calculation and distribution of profit apart from the systems of existing Companies, and to disregard the considerations of all margins until these are specially the subject of remark. In short, it is proposed to consider this subject theoretically, in the hope that thereafter it may be the more easily and the more fittingly applied in a practical manner.

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There is one remark I would desire to make, in the expectation that I may anticipate the point of your criticism. In tracing the nature of profit created under certain supposed deviations from the fundamental data, certain results will, no doubt, be arrived at possessing the peculiar character of these suppositions ; and while these suppositions may be very likely to happen, still there may be in the actual transactions of an Office some other elements mixed up with these natural deviations which will, to a certain extent, destroy the exactness and sharpness of the results arrived at by mathematical investigation, and perhaps render them inapplicable to the full extent to which the assumptions by themselves would seem to justify. There may, for instance, be a profit from surrendered and forfeited policies; from annuities, reversions, sale of consols or property; and when these and similar profits fall into the surplus, the proportion thereof allotted to each policy-holder will not exactly correspond with the amount obtained by a direct calculation based upon the assumptions alone; and accordingly it may be said that these extraneous additions to the surplus may afford a sufficient reason or excuse for the adoption of a simple and, at the same time, a rough and round system of distribution, either by a uniform rate of addition to the sum assured or a uniform rate of diminution from the premiums, or some such general process of distribution where the precision of the exact calculation arrived at under the assumptions hereafter made would be uncalled-for and unnecessary. In reply to such criticism I beg, in the first place, to state, that I would not object to the mere rounding-off of the exact calculation.

The mere giving to each man his own penny is not the object in view. My object is, in the first place, to ascertain what exact calculation has to say on the matter; and, in the second place, to arrive at, if possible, some general process of distribution, simple and practical, and at the same time sufficiently consistent with the results of exact calculation. The addition of an equal bonus, or the diminution of an equal sum from the premium, may equally result from the present investigation; the only stipulation upon which I have to insist being that the mode of distribution *shall in the main* be consistent with the mode in which the profit has been created. I would, in the second place, remark that the profit from surrendered policies may form a very insignificant portion of the total profit. And thirdly, I have considerable difficulty in admitting that the profit derived from reversions or the sale of property, if of large amount compared with that derived from interest and mortality in ordinary assurance transactions, should be carried at once into the divisible surplus. Such transactions cannot always be profitable. They will have their average, no doubt, like ordinary assurances; and being of less frequent occurrence, will be spread over a longer period. And although the general tendency of such transactions, from the mode of calculating the price of reversions, or from the superior knowledge which the Directors and others may possess in the purchase and sale of property, may very frequently be profitable, still I am of the opinion that large profits of the nature of capital ought not to be distributed at once and wholly allotted in respect of past years, but should be spread over a period of years to meet any losses which may yet be sustained on similar transactions.

I shall divide the consideration of the determination and distribution of profit into *three* parts—I. Profit from interest; II. Profit from mortality; and III. Profit from margins.

I. Determination and Division of Profit arising from Interest.

1. *Assurances effected by single payments.*

Taking the case of 5,642 assurances for £100, effected at age 30, by a single contribution of £40·12541, being the sum required by the Carlisle table of mortality at 3 per cent. interest (the single payment representing the extreme case of a large and increasing class of policies wherein the contributions are limited to a term of years), the following will exhibit the

Actual Progress of the Fund, upon the supposition that 4 per Cent. has been realised in the Investment of the Funds.

Year.	Amount of Fund at beginning of each Year.	Interest thereon, at 4 per Cent.	Claims.	Amount of Fund at end of each Year.	Values of the Policies of those remaining Alive.		Difference, or Profit.	Share to each then alive, per £1,000.
					No.	Values.		
	£	£	£	£		£	£	£
1	226,387	9,056	5,700	229,743	5,585	227,480	2,263	4·05
2	229,743	9,190	5,700	233,232	5,528	228,604	4,629	8·37
3	233,233	9,330	5,600	236,963	5,472	229,862	7,101	12·98
4	236,963	9,479	5,500	240,942	5,417	231,258	9,684	17·88
5	240,942	9,638	5,500	245,080	5,362	232,695	12,384	23·10
6	245,080	9,803	5,500	249,383	5,307	234,176	15,207	28·65
7	249,383	9,975	5,600	253,758	5,251	235,601	18,157	34·58
8	253,758	10,150	5,700	258,208	5,194	236,970	21,238	40·89
9	258,208	10,328	5,800	262,736	5,136	238,279	24,457	47·62
10	262,736	10,509	6,100	267,145	5,075	239,327	27,818	54·81

These results, arithmetically derived, are given in the belief that they will afford a better idea of the progressive character of the shares of profit by the several kinds of assurance than would be obtained by the mere inspection of their algebraical formulæ.

If the actual amount of funds at end of each year be divided amongst all then alive, the share to each, expressed by the symbol $W_{x|n}$, would represent their actual interest in the total funds, or the *actual* or the *acquired* values of their policies. And since the amount of funds which were by the fundamental data expected to be on hand, divided by the number alive, would give the fundamental or expected value of the policy, and expressed by the letter $V_{x|n}$, the difference $(W_{x|n} - V_{x|n})$ is the amount or share of profit possessed by each. Thus :—

Years.	Share of actual Funds to each alive. $W_{x\|n}$		Share of expected Funds to each alive. $V_{x\|n}$		Difference—being Profit to each then alive. $Profit_{x\|n}$
1 . .	£411·35	—	£407·30	=	£4·05
2 . .	421·91	—	413·54	=	8·37
3 . .	433·05	—	420·07	=	12·98
4 . .	444·79	—	426·91	=	17·88
5 . .	457·07	—	433·97	=	23·10
6 . .	469·91	—	441·26	=	28·65
7 . .	483·26	—	448·68	=	34·58
8 . .	497·13	—	456·24	=	40·89
9 . .	511·56	—	463·94	=	47·62
10 . .	526·39	—	471·58	=	54·81

These results are much more readily obtained by means of algebraical formulæ.

Let A_x represent the single payment paid at entry, and A'_x

the single premium calculated at the rate of interest actually experienced; also, N_x, D_x, M_x, &c., having their usual meanings, let the same quantities calculated at the experienced rate of interest be denoted by N'_x, D'_x, M'_x, &c.

Then, since the value of a policy implies the accumulation of the premiums at the actual rates of mortality and interest experienced $= A_x \cdot \frac{D'_x}{D'_{x+n}}$, less the accumulated amount of the actual claims $= \frac{M'_x - M'_{x+n}}{D'_{x+n}}$; therefore the actual or the acquired value of the policy at end of n years $= \frac{A_x D'_x - M'_x + M'_{x+n}}{D'_{x+n}}$.

This sum represents the *share of the actual funds* belonging to each survivor of n years; and since the share of the expected amount of funds, or the fundamental value of his policy, is represented by A_{x+n}, the difference between these will express the share of profit created to the end of n years, and belonging to each survivor at that time: thus—

Profit derived in n years from a policy effected at the age x

$$= \frac{A_x D'_x - M'_x + M'_{x+n}}{D'_{x+n}} - A_{x+n}.$$

The profit is thus seen to be the difference between two values of the same policy; the one value being the accumulated amount of the residue of the premium (that which remains on hand each year after payment of the previous year's claim), at the *profitable* rate of interest, the 4 per cent.; and the other value being the accumulated amount of the same residue at the fundamental or the profitless rate of interest, the rate which *excludes* the profit, the 3 per cent. The difference between these values is manifestly the amount of the profit belonging to each survivor of n years.

The above expression for the profit may be transformed into the following:—

$$\frac{A_x D'_x}{D'_{x+n}} - \frac{A'_x D'_x}{D'_{x+n}} + A'_{x+n} - A_{x+n}$$

$$= (A_x - A'_x)\frac{D'_x}{D'_{x+n}} - (A_{x+n} - A'_{x+n}).$$

The formula for the first year's profit,

$$= \frac{A_x D'_x}{D'_{x+1}} - \frac{M'_x}{D'_{x+1}} + \frac{M'_{x+1}}{D'_{x+1}} - A_{x+1}$$

$$= \frac{A_x D'_x}{D'_{x+1}} - \left(\frac{M'_x}{D'_{x+1}} - \frac{M'_{x+1}}{D'_{x+1}}\right) - \frac{A_x D_x}{D_{x+1}} + \frac{M_x - M_{x+1}}{D_{x+1}}$$

$$= \frac{A_x v'^x l_x}{v'^{x+1} l_{x+1}} - \frac{v'^{x+1} d_x}{v'^{x+1} l_{x+1}} - \frac{A_x v^x l_x}{v^{x+1} l_{x+1}} + \frac{v^{x+1} d_x}{v^{x+1} l_{x+1}}$$

$$= \frac{A_x l_x}{l_{x+1}}\left(\frac{1}{v'} - \frac{1}{v}\right) = \frac{A_x l_x}{l_{x+1}}\left\{(1+i') - (1+i)\right\} = \frac{A_x \Delta i}{p_x}.$$

And, similarly, it may be shown that the profit in one year on a policy effected at age $x+n$ is

$$\frac{\Delta i A_{x+n}}{p_{x+n}}.$$

The profit for the first period of, say 5 years, is

$$\frac{A_x - A'_x}{p_{x|5} v'^5} - (A_{x+5} - A'_{x+5});$$

and for the second period of 5 years, on a policy opened at x=the profit created in the first 5 years of a policy opened at age $x+5$, and similarly for other periods.

The following are illustrations of the amount of profit from 1 per cent. interest created in each period of 5 years on single payment policies, and serve also to show the profit on policies opened at each of these ages.

	Per £1,000 Policy.		Per £1,000 Policy.
30 to 35 . .	£23·09	60 to 65 . .	£41·66
35 to 40 . .	25·13	65 to 70 . .	45·78
40 to 45 . .	27·50	70 to 75 . .	54·20
45 to 50 . .	29·72	75 to 80 . .	64·66
50 to 55 . .	32·71	80 to 85 . .	77·17
55 to 60 . .	36·82		

From which it will be seen, that profit from interest on assurances effected by single payment seems to possess the character of being nearly equal each year, increasing to a larger amount in the later years of life, and that, accordingly, the usual bonus additions of an equal percentage per annum would be in fair consistency therewith.

If the valuation of the policy, in respect of the future risk, be estimated at the experienced rate of interest—the 4 per cent.—the *first* profit on the policy will consist of the entire difference between the single premiums at 3 and at 4 per cent. accumulated.

For if in the formula

$$(A_x - A'_x)\frac{D'_x}{D'_{x+n}} - (A_{x+n} - A'_{x+n}),$$

we have $$A_{x+n} = A'_{x+n},$$

the profit becomes

$$=(A_x - A'_x)\frac{D'_x}{D'_{x+n}};$$

and therefore, if this profit be appropriated at a *first* investigation at end of n years, the future profit will be nothing; because the estimate of the future experience will be based upon the 4 per cent., the rate at which it is assumed the funds will improve.

The difference between the net premiums at 3 and at 4 per cent. at age x, is the present value of all the future profit which will be created.

2. *Assurances effected by annual premiums.*

Passing now to the consideration of assurances effected by annual premiums, I shall likewise assume that the premiums charged for the assurance are based upon Carlisle 3 per cent., that the experienced rate of interest is 4 per cent., and that the "valuations" are based upon the fundamental rate of 3 per cent.

As I shall, on some future occasion, probably make reference to the figures in the expected and in the actual progress of such a fund, there is exhibited the following view of

1. *The Expected Progress of the Fund.*

5,642 Assurances at Age 30 (Carlisle 3 per Cent.); Premium = £19·5192 per £1,000.

Year.	Funds at beginning of each Year.	Interest, at 3 per Cent.	Claims.	Funds at end of each Year, with next Year's Renewals.	Values of Policies of those Alive.		Profit.
					No.	Values, by an actual Calculation.	
	£	£	£	£		£	£
1	110,127	3,304	57,000	56,431 109,014	5,585	56,429	0
2	165,445	4,963	57,000	113,408 107,902	5,528	113,408	0
3	221,310	6,639	56,000	171,949 106,809	5,472	171,949	0
4	278,758	8,363	55,000	232,121 105,735	5,417	232,122	0
5	337,856	10,136	55,000	292,992 104,662	5,362	292,993	0
6	397,654	11,929	55,000	354,583 103,588	5,307	354,584	0
7	458,171	13,745	56,000	415,916 102,495	5,251	415,918	0
8	518,411	15,552	57,000	476,964 101,383	5,194	476,964	0
9	578,347	17,350	58,000	537,697 100,250	5,136	537,696	0
10	637,947	19,138	61,000	596,086	5,075	596,085	0

The following is a view of

2. *The Actual Progress of the Fund.*

Experience = Carlisle 4 per Cent.; Expectations, Carlisle 3 per Cent.
5,642 Assurances for £1,000, at Age 30.

Year.	Funds at beginning of each Year.	Interest at 4 per Cent.	Claims.	Funds at end of each Year and Renewals.	Values of Policies of those Alive.		Total Profit.	Share to each per £1,000.
					No.	Values.		
	£	£	£	£		£	£	£
1	110,127	4,405	57,000	57,532 109,014	5,585	56,429	1,103	·197
2	166,546	6,662	57,000	116,208 107,902	5,528	113,408	2,800	·506
3	224,110	8,964	56,000	177,074 106,809	5,472	171,949	5,125	·937
4	283,883	11,355	55,000	260,239 105,735	5,417	232,122	8,117	1·498
5	345,974	13,899	55,000	304,813 104,662	5,562	292,993	11,820	2·204
6	409,475	16,379	55,000	370,854 103,588	5,507	354,584	16,270	3·066
7	474,442	18,978	56,000	437,419 102,495	5,251	415,918	21,501	4·093
8	539,914	21,596	57,000	504,511 101,383	5,194	476,964	27,547	5·304
9	605,894	24,236	58,000	572,130 100,250	5,136	537,696	34,434	6·701
10	672,380	26,895	61,000	638,275	5,075	596,085	42,190	8·317

The same results may be brought out by means of formulæ.

The value of the policy based upon the elements or data experienced, after the form of the retrospective value of the policy,

$$W_{x|n}=\pi_x \frac{N'_{x-1}-N'_{x+n-1}}{D'_{x+n}} - \frac{M'_x-M'_{x+n}}{D'_{x+n}};$$

and the value of the same policy based upon the data which it is assumed will be experienced in future years, that is, upon the same data as those inherent in the calculation of the premiums, the Office being supposed to be as willing to renew the risk upon these data as it is to effect a new assurance upon the same data, (an investigation consisting merely in reassuring the same lives on the original data, and the value of the policy *reserved* being in supplement of the difference between receiving the premium at the younger age and that required for a new assurance at the older age,) $=V_{x|n}$. The difference between these two values will show the amount of the profit which has accrued in the past n years :—

$$W_{x|n}-V_{x|n}=\frac{\pi_x(N'_{x-1}-N'_{x+n-1})-M'_x+M'_{x+n}}{D'_{x+n}}-V_{x|n}.$$

The profit created in the first 5 years,

$$\frac{\pi_x(N'_{x-1}-N'_{x+4})-M'_x+M'_{x+5}}{D'_{x+5}}-V_{x|5};$$

and in the second 5 years,

$$\text{Profit}_{x|10}-\text{Profit}_{x|5}\frac{D'_{x+5}}{D'_{x+10}};$$

and in m years after n years,

$$=\text{Profit}_{x|n+m}-\text{Profit}_{x|n}\frac{D'_{x+n}}{D'_{x+n+m}}.$$

The following shows the amount of the profit under the form of "the difference between two values of the same policy"; also the amount created each 5 years — the previous profits having been distributed:—

Year. (n.)	Actual or acquired Value of the Policy. $W_{x\|n}$.	Fundamental Value of the Policy. $V_{x\|n}$.	Profit $_{x\|n}$. per £1,000.	Quinquennial Profit per £1,000.
1	10·301	10·104	·197	
2	21·021	20·515	·506	
3	32·360	31·423	·937	
4	44·349	42·851	1·498	
5	56·846	54·642	2·204	2·204
6	69·880	66·814	3·066	
7	83·301	79·208	4·093	
8	97·134	91·829	5·304	
9	111·391	104·691	6·701	
10	125·772	117·455	8·317	5·484
15	199·63	179·69	19·94	9·076
20	294·50	255·62	38·88	12·799
25	416·40	347·73	68·67	17·604
30	557·95	440·99	116·96	23·550
35	719·15	515·57	201·58	29·810
40	948·89	604·84	344·05	35·770
45	1328·50	683·22	645·28	45·280
50	2575·40	739·03	1436·36	56·460
	&c.	&c.	&c.	&c.

If the valuation of the policy in respect of the future be based upon the rate which has been experienced, under the supposition that that rate will be obtained during the entire existence of the policy, the profit will consist of the accumulated difference between the annual premium charged and that required for the risk at the experienced rate of interest:—

$$W_{x|n}=\frac{\pi_x(N'_{x-1}-N'_{x+n-1})-M'_x+M'_{x+n}}{D'_{x+n}};$$

and if $V_{x|n}$ be assumed to be based upon the experienced data,

$$= \frac{a'_x - a'_{x+n}}{1+a'_x} = \frac{M'_{x+n}}{D'_{x+n}} - \pi'_x \frac{N'_{x+n-1}}{D'_{x+n}}$$

$$W_{x|n} - V_{x|n} = (\pi_x - \pi'_x) \frac{N'_{x-1} - N'_{x+n-1}}{D'_{x+n}}.$$

In this view the following will exhibit the amount of the profit to the end of n years; also the amount of the profit created in each period of 5 years:—

Year. (n)	Profit to end of n Years.	Quinquennial Profit.	Year. (n)	Profit to end of n Years.	Quinquennial Profit.
1	2·065		15	45·727	11·577
2	4·234		20	71·379	11·571
3	6·513		25	105·380	11·629
4	8·906		30	155·26	11·920
5	11·423	11·423	35	240·51	12·490
6	14·067		40	380·62	12·820
7	16·825				
8	19·784				
9	22·875				
10	26·144	11·461			

According to this view, the quinquennial profits will scarcely afford an equal quinquennial bonus during life. The yearly profit being of equal amount will certainly *not* afford a bonus of increasing value.

And if the future premiums π_x be valued by the annuity at the experienced rate of interest, the profit will be $(\pi_x - \pi'_x) \frac{N'_{x-1}}{D'_{x+n}}$, being the present value of the difference between the premium charged and that required, accumulated to the end of n years. Thus—

$$\frac{\pi_x(N'_{x-1} - N'_{x+n-1}) - M'_x + M'_{x+n}}{D'_{x+n}} + \pi_x \frac{N'_{x+n-1}}{D'_{x+n}} - \frac{M'_{x+n}}{D'_{x+n}} = \frac{\pi_x N'_{x-1} - M'_x}{D'_{x+n}}$$

$$= (\pi_x - \pi'_x) \frac{N'_{x-1}}{D'_{x+n}}.$$

I now propose, when the valuation proceeds upon the original or fundamental data, that the total surplus, assuming it to arise entirely from interest, be distributed in proportion to the ratios or measures formed upon the difference between the retrospective and the prospective values of the policy:—

On annual payment policies—

$$W_{x|n} - V_{x|n} = \frac{\pi_x(N'_{x-1} - N'_{x+n-1}) - M'_x + M'_{x+n}}{D'_{x+n}} - V_{x|n};$$

On single payment policies—

$$W_{x|n}-V_{x|n}=(A_x-A'_x)\frac{D'_x}{D'_{x+n}}-(A_{x+n}-A'_{x+n}),$$

and similarly ascertained ratios for other scales of premium.

And when the valuation proceeds upon the experienced data, I would propose that the total surplus be distributed upon some principle which shall have reference to the entire future duration of the policy:—

Total surplus, being the profit accrued to end of n years, and the present value of all future profit on annual premium policies . $=(\pi_x-\pi'_x)\frac{N'_{x-1}}{D'_{x+n}}$.

Ditto on single payment policies . . . . $=(A_x-A'_x)\frac{D'_x}{D'_{x+n}}$.

* * * * *

The principle of the measures of distribution will at once be seen to be applicable to all kinds of policies.

The author then considers a method of approximation to the above results, supposing that the single premiums received are accumulated at compound interest. He says—Taking first the case of *single payment policies,* it has been shown that the profit is the difference between the A_x accumulated at mortality and the profitable rate of interest, less the same sum accumulated at mortality and the fundamental rate of interest, less the difference in the accumulated claims. Excluding the mortality, this becomes (i' being the profitable, and i the profitless rate of interest)—

$$\text{Profit}_{x|n}=A_x\{(1+i')^n-(1+i)^n\};$$

the first quinquennial profit would be

$$A_x\{(1+i')^5-(1+i)^5\};$$

and the second would be

$$A_{x+5}\{(1+i')^5-(1+i)^5\}.$$

The values of these, compared with the former ratios of a single payment assurance for £1,000, effected at 30, are here given:—

1st five years	=23·02,	while the former value is		=23·09
2nd	„ =24·90,	„	„	25·13
3rd	„ =27·06,	„	„	27·50
4th	„ =29·20,	„	„	29·72
5th	„ =31·80,	„	„	32·71
6th	„ =34·97,	„	„	36·82
7th	„ =38·18,	„	„	41·66
8th	„ =40·80,	„	„	45·78
9th	„ =43·80,	„	„	54·20

Annual payment policies.—In the same way, the approximate ratio will be the difference between the accumulated premiums at the profitable and at the profitless rates of interest—

$$\text{Profit}_{x|5}=\pi_x\left\{\frac{(1+i')^5-1}{i'}(1+i')-\frac{(1+i)^5-1}{i}(1+i)\right\},$$

and the second,

$$V_{x|5}\{(1+i')^5-(1+i)^5\}+\pi_x\left\{\frac{(1+i')^5-1}{i'}(1+i')-\frac{(1+i)^5-1}{i}(1+i)\right\}$$

$$=V_{x|5}\Delta(1+i)^5+\pi_x\Delta(1+i)\frac{(1+i)^5-1}{i}$$

&c. &c.

The values of these, compared with the values of the ratios deduced from the actuarial formulæ, are—

1st five years	=	£3·21,	while	£2·20	= value formerly deduced.	
2nd „	=	6·34,	„	5·48	„	„
3rd „	=	9·95,	„	9·08	„	„
4th „	=	13·52,	„	12·80	„	„
5th „	=	17·87,	„	17·60	„	„
6th „	=	23·16,	„	23·55	„	„
7th „	=	28·52,	„	29·81	„	„
8th „	=	32·91,	„	35·77	„	„
9th „	=	37·92,	„	45·28	„	„

These approximations will be considered amply sufficient substitutes for the more correct ratios.

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In regard to the two modes of valuation previously referred to, that on the net and that on the experienced data, it appears to me to be uncalled for to state which of these modes I would advocate. Both are good, and, under different circumstances, one is to be preferred to the other. These circumstances I am not now considering. I am merely setting forth what occurs to me to be the essential arrangements of each method, the measures of distribution of the profit calculated to have arisen by a valuation on the net data, and the system of distribution when the valuation of the assurance and the premium payable is based upon the experienced data. My views in general upon the system of bonuses, may, perhaps, be gathered from the following. If we suppose the scale of premiums (all margins being excluded) to be based upon the "A table of mortality, at i per cent," insuring the minimum sum of £100, and the experience to be represented by the "A′ table, at i' per cent," upon which scale the premium by the former will assure at age x, say, somewhere about £125,—I would prefer that the sums assured should be between the £100 and the £125, not only to those who only lived one year, but also to those who lived

to old age. Such a system carries out to the full the principle of an average—the principle upon which life assurance is founded. There is, by all the ordinary bonus systems, a certain amount of injustice done to the assured dying in the early years after having paid premiums which, on a retrospect of the experience, would have provided the equal average amount of £125 to all the assured, whether they died in the first year of their policies, or lived to the extreme years of life, but who received the sum of £100 only. I am not considering whether such a scheme is practicable or impracticable, or whether it would be appreciated by a public, now so long accustomed to bonuses increasing with the years of the policy, or to what extent it would be expedient to approximate to such a system; but it seems to me that such a scheme, or some modification of it, would carry out the principle of life assurance to a much greater extent than either of the two systems of bonus distribution which have been under consideration.

On the Metric System of Weights and Measures, and its proposed Adoption in this Country. *By* SAMUEL BROWN, F.S.S., *Vice-President of the Institute of Actuaries.*

THE great advantages, social, commercial, and political, which would attend the use of one system of weights, measures, and coins throughout the world, have generally been admitted, but as generally considered impossible. Such a result is frequently deemed to be merely the dream of a visionary, or the speculation of a philosopher, who has no practical knowledge of the world, and is incapable of appreciating the difficulties which stand in the way of accomplishing so desirable an object. It must be granted that the difficulties are great. There are prejudices to be overcome, ignorance to be enlightened, national pride to be vanquished, and, in many cases, trade customs, the growth of ages, to be abolished, before one nation can combine with another, much less several nations agree together, to adopt a common system. Yet, in spite of all these obstacles, the present century has witnessed such great changes in the old practice, and all tending towards a uniform standard, that I propose to give a brief sketch of the present position of this question, and to show what has been done and is doing to carry on a work so important to the social condition of any country, and to the free interchange of its produce or manufacture with other countries.